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**AN ANALYTICAL STUDY OF THE EFFECT OF COOLANT FLOW  
VARIABLES ON THE KINETIC ENERGY OUTPUT  
OF A COOLED TURBINE BLADE ROW**

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AN ANALYTICAL STUDY OF THE EFFECT OF COOLANT FLOW VARIABLES ON THE KINETIC ENERGY  
OUTPUT OF A COOLED TURBINE BLADE ROW

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Abstract

The study shows that the change in output of a cooled turbine blade row relative to the specific output of the uncooled blade row can be positive, negative, or zero depending on the velocity, injection location, injection angle, and temperature of the coolant.

Comparisons between the analytical results and experimental results for four different cases of coolant discharge, all at a coolant temperature ratio of unity, show good agreement for three cases and rather poor agreement for the other.

To further test the validity of the method, more experimental data is needed, particularly at different coolant temperature ratios.

Introduction

It is known that the work output of a cooled turbine is affected by the amount, velocity, injection location, injection angle, and temperature of the coolant flow introduced in the blade rows, but the effect of each of these variables on blade row performance has not been well defined.

Accordingly an analytical study has been made in an effort to determine the effect of these coolant variables on blade row output. This study is part of a continuing program at the NASA Lewis concerning the effect of coolant flow on turbine performance. The results of some parts of this program which have been completed are reported in Refs. 1 to 9. References 1 to 3 report the results of experimental and analytical investigations of the effects of turbine stator blade trailing-edge coolant ejection on turbine stator and stage performance; and Refs. 4 to 7 report the results of experimental and analytical investigations of the effects of two types of stator blade transpiration coolant discharge on turbine stator and stage performance. Reference 8 summarizes the results of Refs. 1 to 7. The main conclusions of these reference investigations were that coolant flow ejected from the trailing edge parallel to the main stream contributed significantly to the turbine stage work output; whereas, coolant flow ejected through a porous skin covering the complete stator blade surface contributed little or nothing to the turbine stage work output.

In Ref. 9, analytical methods for determining the effect of coolant air on turbine efficiency are described. In these analyses, the effects on blade row output of multiple coolant ejection with different coolant variables were considered in their aggregate by use of single valued coolant pressure coefficients, the coolant pressure coefficient being defined as the ratio of the dynamic head of the coolant flow to the dynamic head of the primary flow at blade outlet. Since, for purposes of the reference analyses, these pressure coefficients could be assumed, no means were provided for their

determination.

The analyses of this paper may be considered a continuation of that of Ref. 9. It describes an analytical method for predicting the effect of specific coolant variables on blade row output. Charts based on the method are presented which permit rapid estimation of the effect on blade row output of coolant ejection from single or multiple locations with different coolant variables. From these results, overall values of blade row coolant pressure coefficients can be obtained if desired for use in Ref. 9 to compute the effect of the total coolant flow on turbine efficiency.

Examples of use of the method for actual blade rows are presented, and results from the method are compared with experimental results.

Results of the study are presented in terms of change in kinetic energy output relative to that of the uncooled blade row.

Analyses

The general procedure in the analyses is to determine the change in uncooled blade row output due to coolant addition. To accomplish this, the effective kinetic energy and momentum outputs of the coolant and primary (uncooled) flows are first determined. Then the two flows are mixed and the output of the mixed flow determined. Knowing the output of the cooled and uncooled flows, the change in output of the uncooled blade row due to coolant addition is obtained.

A discussion of the analyses of the problem is presented in the following three sections. (The mathematical expressions corresponding to the analyses are in Appendix B. Symbols are in Appendix A.) The first section concerns how the different coolant flow variables affect the coolant flow output. The second section concerns how the coolant flow output affects the mixed flow output. And the third section concerns how trailing-edge coolant ejection, which is a special case of coolant discharge, affects the mixed flow output.

For purposes of simplification, the analyses assume that the expansion efficiency of the coolant and primary flows are the same. The use of this assumption makes the computations independent of efficiency, thus isentropic processes can be used in the analyses. The analyses also considers the efficiency of the primary flow before mixing with the coolant flow to be the same as the uncooled blade row efficiency. Finally, the analyses assume that the coolant and primary flows mix at constant pressure equal to blade row exit static pressure. Then  $p_2 = p_{p,2} = p_m$ .

Coolant Flow Output

As indicated in Fig. 1, the kinetic energy and momentum of the coolant flow at blade row exit are

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dependent upon the temperature, location, velocity, and angle at which the coolant flow is discharged into the main stream. Four representative cases of coolant discharge that may occur are shown.

In case 1, the coolant is ejected on the pressure side of the blade in a direction normal to the primary flow. For this case, the kinetic energy of the entering coolant is considered to be lost by throttling and so is not available for useful blade row output. However, as indicated by the blade-surface-pressure diagram, the static pressure of the coolant is larger than the blade exit static pressure. For this case, as indicated by the HS diagram, the coolant flow then has specific kinetic energy at blade row exit corresponding to the enthalpy drop between its static discharge pressure and blade exit static pressure.

Case 2 is the same as case 1 except the coolant is discharged with a component of velocity in the direction of primary flow instead of normal to the primary flow. As indicated on the HS diagram, the component of kinetic energy parallel to the primary flow represents useful kinetic energy. So the total specific kinetic energy output of the coolant flow for case 2 is the sum of the specific kinetic energies represented by the energy component parallel to the primary flow and the specific kinetic energy output corresponding to the enthalpy drop between the blade surface static pressure and blade exit static pressure.

Case 3, as shown on Fig. 1, is for coolant discharge normal to the primary flow from the diffusion region on the suction surface of the blade. For this case, as indicated by the blade surface-pressure diagram and the HS diagram, the static pressure of the coolant at the location of discharge on the blade surface is less than the blade exit static pressure. There is then no enthalpy drop to blade exit for conversion to kinetic energy. Instead energy must be added to the coolant flow to increase the coolant pressure from discharge static pressure to blade exit static pressure. The compression work required for this pressure increase, of course, causes a decrease in blade row output. As indicated by the HS diagram, it is assumed in these analyses that the specific work required for this pressure increase is that of isentropic compression.

Case 4 is the same as case 3, except that the direction of coolant discharge is not normal to the primary flow. For this case, again as the HS diagram suggests, the analyses assume that the coolant is first compressed to blade exit static pressure, and during the compression, the component of coolant energy parallel to the main stream is conserved. For case 4, the net specific kinetic energy output of the coolant at blade row exit is then the sum of the equivalent specific kinetic energy required for compressing the coolant to stage exit static pressure, which is negative, and the useful specific kinetic energy represented by the energy component parallel to the main flow.

Summarizing this section, the effects of coolant flow variables on the specific kinetic energy of the coolant flow at blade exit before mixing have been discussed. Depending on the coolant variables, the specific kinetic energy of the coolant before mixing was found to vary from

positive to negative values. The coolant conditions which determine these variations in energy can be easily identified. First the total effective pressure of the coolant relative to the blade row exit static pressure determines whether the energy of the coolant is positive or negative. If  $p_{c,h}$  is less than  $p_2$  the energy is negative, and if  $p_{c,h}$  is greater than  $p_2$ , the energy is positive.

It is also known, as the HS diagrams imply, that for a given value of positive head, the output of the coolant flow increases with increased coolant temperature, and that for a given value of negative head, the required coolant compression work increases with increased coolant temperature. For these reasons, as will be shown in the Results section, the change in output of the uncooled blade row for a given coolant mass flow can be correlated by the temperature ratio of the primary and coolant flows,  $T_p/T_{c,h}$ , and a coolant flow pressure coefficient  $k_p$  which is the ratio of the total dynamic head of the coolant flow relative to the total dynamic head of the primary flow. Thus,  $k_p = (p_{c,h} - p_2)/(p_p - p_2)$ . Also since the ratios of coolant flow specific kinetic energy to primary flow specific kinetic energy at blade row exit,  $(V_c/V_p)_2^2$  and  $(V_{c,pp}/V_{p,2})^2$  are dependent upon the coolant temperature ratio and pressure coefficient, the change in blade row output due to coolant flow can be correlated by the parameters  $(V_c/V_p)_2$  and  $(V_{c,pp}/V_{p,2})$ .

#### Effect of Coolant Flow on Mixed Flow Output

These analyses consider the mixed flow output to be affected by coolant flow in two ways. The first way is that some of the kinetic energy of the primary and coolant flow is lost as a result of momentum exchange during mixing. The second way is that the mixed flow must contribute compression work to increase the pressure of any coolant flow discharged in the diffusion region on the blade surface.

These analyses assume that the momentum exchange between the primary and coolant flows occur after the two streams exit from the blade row and that the exchange occurs at constant pressure equal to blade row exit static pressure. The energy loss due to momentum exchange during mixing is then dependent on the difference in momentum between the primary and coolant flows at blade row exit.

Further, in these analyses, the effect on blade row output of any compression work required because of coolant discharge in a diffusion region is determined by simply subtracting the isentropic compression work from the blade row output occurring after the momentum exchange.

#### Effect of Trailing-Edge Coolant Discharge on Blade Row Output

There are two major effects on blade row output resulting from trailing-edge discharge of coolant flow. These two effects are indicated in Fig. 2. As indicated in Fig. 2(a), one effect is that the discharge of coolant flow parallel to the main gas stream contributes useful kinetic energy to the blade row output. The other effect, as indicated by Figs. 2(a) and (b), is that the flow of coolant from the trailing edge reduces the momentum deficit occurring in the trailing-edge region and

thus reduces the trailing-edge loss that occurs in the absence of coolant flow. To determine the effect of coolant flow on trailing-edge loss then requires that the trailing-edge loss with and without coolant flow be determined.

In this analysis, the trailing-edge loss without coolant flow was determined from the method of Ref. 11 assuming that the trailing-edge geometry was that of the cooled blade without the slot. The trailing-edge loss with coolant flow was determined using the assumption that the trailing-edge loss is related to the momentum deficit of the coolant flow relative to the primary flow, the loss being minimum when the velocity of the primary and coolant flows are equal and maximum when the velocity of the coolant is zero. This effect of coolant flow on trailing-edge loss is indicated in Fig. 3(a) for a trailing-edge slot having very thin walls. As shown in the figure, the trailing-edge loss would be maximum for zero coolant flow and equal to zero when the velocity of the coolant flow equals that of the primary flow. As indicated in Fig. 3(b), with finite slot wall thickness, the loss would not reach zero when the coolant and primary flow velocities were equal since the thickness of the slot walls cause loss at this condition. Also as indicated, the loss due to finite slot walls influence the straight line relationship of loss vs  $(V_{c,sl}/V_{p,2})$  which occurs with very thin slot walls. The exact effects of different trailing-edge slot geometries on trailing-edge loss are considered in the mathematical expressions of Appendix B and the prediction method of Appendix C.

### Results

The results of the analyses of the effect of coolant flow on blade row output are presented in three parts. In the first part the effect of coolant discharge normal to the blade surface from all locations on the blade except the trailing edge are discussed. Next, the effect of coolant discharge angle and discharge velocity are discussed. Then the effect of trailing-edge ejection, which is a special case of coolant flow discharge, is presented.

The results of the effects of coolant variables on blade row output are presented in terms of percent change in kinetic energy output relative to the specific kinetic energy output of the uncooled blade row. Thus  $\Delta h/h_o \times 100 = [(V_m^2(1+y) - v_o^2)/v_o^2] \times 100$ .

#### Effect of Coolant Discharge Normal to the Blade Surface

In Fig. 4, the effect of coolant discharge normal to the blade surface on blade row output is presented as a function of the parameters  $(V_c/V_{p,2})$  and  $(V_{c,pp}/V_{p,2})$ .

The values of  $(V_c/V_{p,2})$  shown represent conditions when the effective coolant discharge pressure on the blade surface is greater than blade exit static pressure, and the values of  $(V_{c,pp}/V_{p,2})$  shown represent conditions when the coolant discharge pressure on the blade surface is less than blade exit static pressure. Values of  $(V_c/V_{p,2})$  are the ratio of the coolant flow velocity to the primary flow velocity at blade row exit and

indicate the relative blade row specific energies of the coolant flow and primary (or uncooled) flows. The values of  $(V_{c,pp}/V_{p,2})$  shown do not occur as such in the blade row but, as discussed in the Analyses section, are used to represent the specific energy, relative to the specific kinetic energy of the primary flow, that is required to compress the coolant from its discharge pressure to blade row exit static pressure. The values of  $(V_{c,pp}/V_{p,2})$  are then defined as the negative value of the ratio of the isentropic velocity between blade exit static pressure and the coolant discharge pressure to the velocity of the primary flow at blade row exit.

As indicated by the dashed and solid lines on the figure, the results are dependent on the coolant fraction  $y$  used to compute the results. Since the percent coolant discharge from a given location is considered likely to be much closer to one percent than 10 percent (or 50 percent), the remaining results are presented on the basis of one percent discharge from a given location.

The results of Fig. 4 clearly show that, for the assumptions of these analyses, the fundamental parameters affecting blade row output are  $(V_c/V_{p,2})$  and  $(V_{c,pp}/V_{p,2})$ . The values of these parameters are, of course, dependent upon the other coolant flow variables. The effect of the other coolant variables will be considered later in this section.

The results of Fig. 4 show that the potential output of the coolant continually decreases as  $(V_c/V_{p,2})$  and  $(V_{c,pp}/V_{p,2})$  decrease. This is expected since the energy of the coolant relative to the energy of the primary flow is decreasing as these parameters decrease. Some particular results on Fig. 4 are worth noting. When  $(V_c/V_{p,2})$  equals unity, the increase in output per percent coolant flow is, of course, one percent because the specific kinetic energy of the coolant flow equals the specific kinetic energy of the primary flow, and there is no kinetic energy loss due to momentum exchange after mixing. Continuing for decreasing values of  $(V_c/V_{p,2})$ , when  $(V_c/V_{p,2})$  equals .5, the net output of the coolant flow is zero, and when  $(V_c/V_{p,2})$  equals zero, the loss in output relative to the specific output of the uncooled blade row is essentially one percent for each percent of coolant mass flow added to the blade row.

The loss of 1.0 percent per percent coolant for a  $(V_c/V_{p,2})$  equal to zero occurs because of the following. In the mixing process, the specific kinetic energy of the mixed flow is reduced relative to the primary flow by very nearly two percent for each percent of coolant flow. A net loss in output relative to the uncooled blade row of essentially one percent per percent coolant flow then results because the mixed flow is one percent larger than the primary flow.

For a value of  $(V_c/V_{p,2})$  equal to zero, the effective coolant discharge pressure is, of course, equal to the blade row exit static pressure. As the coolant discharge pressure is decreased to values less than the blade row exit static pressure, the values of  $(V_{c,pp}/V_{p,2})$  apply to the coolant conditions, and the loss in specific output of the uncooled blade row continues to increase as  $(V_{c,pp}/V_{p,2})$  decreases because of the required increase in coolant compression work.

Having considered the effects of the fundamental coolant flow parameters on blade row output, the effects of the coolant variables, which determine these parameters, will now be considered. The coolant variables which determine  $(V_c/V_p)_2$  and  $(V_{c,pp}/V_p)_2$  are coolant temperature ratio  $T_p/T_{c,h}$ , coolant pressure coefficient  $k_p$ , and the ratio of primary air blade exit static pressure to primary air inlet total pressure  $p_2/p_p$ . The coolant pressure coefficient  $k_p$  is the ratio of the pressure difference between the effective pressure of the coolant flow and blade exit static pressure to the pressure difference between the total pressure of the primary flow and blade exit static pressure. Thus,  $k_p = (p_{c,h} - p_2)/(p_p - p_2)$ , which indicates for a given coolant temperature ratio, the relative energies of the coolant and primary flows.

Shown on Fig. 5, then are the effects of coolant flow on blade row output as a function of the independent coolant variables; coolant temperature ratio, coolant pressure coefficient, and primary air pressure ratio. Also superimposed on the figure are values of  $(V_c/V_p)_2$  and  $(V_{c,pp}/V_p)_2$  to show the interrelation between the independent coolant flow variables and these fundamental coolant parameters.

The results on the figure show that the coolant temperature ratio and the coolant pressure coefficient have a large effect on the coolant flow output and that for positive values of  $k_p$ , the effect of primary air pressure ratio is largely accounted for by  $k_p$ , while for negative pressure ratios, it is not entirely accounted for by  $k_p$ . The general results on Fig. 5 show that, other coolant variables being constant, the useful blade row output decreases with decreasing  $k_p$ . For positive values of  $k_p$ , the results also show, other things being the same, that the useful output decreases with decreasing coolant temperature due to the decreasing kinetic energy of the coolant flow. For negative values of  $k_p$ , the results show that, with other variables fixed, the useful output increases with decreasing coolant temperature. This is due to the decrease in required coolant flow compression work with decreasing coolant temperature.

An interesting result shown on Fig. 5 is that, for a coolant temperature ratio of four, the addition of coolant flow causes a loss in specific output relative to the uncooled blade row for all values of  $k_p$  less than 1.0. These results occur because of the temperature ratio effect. With large values of temperature ratio, even with large values of  $k_p$ , the velocity and kinetic energy of the coolant flow relative to the primary is small. As a result of the low relative velocity of the coolant flow, the kinetic energy of the coolant flow before mixing is more than nullified by the reduction in specific kinetic energy resulting from the mixing process.

#### Effect of Coolant Discharge Velocity and Angle

The preceding discussion has shown the effects of coolant temperature ratio, coolant pressure coefficient, and primary-air pressure ratio on coolant flow output. The separate effects of coolant discharge velocity and discharge angle, which are hidden in the effect of  $k_p$ , have yet to be considered.

The separate effects of coolant discharge velocity and angle on blade row output are presented in Fig. 6. These results are shown as a function of the parameters  $(V_c/V_p)_2$ ,  $(V_{c,pp}/V_p)_2$ , and  $(V_{c,h} \cos \alpha/V_p)_2$ . The parameters  $(V_c/V_p)_2$  and  $(V_{c,pp}/V_p)_2$  have been previously defined, and the parameter  $(V_{c,h} \cos \alpha/V_p)_2$  is the ratio of the coolant velocity component parallel to the main stream at the coolant discharge location to the velocity of the primary air at blade row exit.

The results show, as expected, that increasing the coolant velocity component parallel to the main stream improves the blade row output for all conditions. The results also show that for a given  $(V_{c,h} \cos \alpha/V_p)_2$  the improvement in blade row output increases with decreasing  $(V_c/V_p)_2$ , reaching a maximum value at a  $(V_c/V_p)_2$  of zero and then remaining constant for all values of  $(V_{c,pp}/V_p)_2$ . This result may be explained as follows. When the coolant is ejected at an angle to the flow, the parameter  $(V_{c,h} \cos \alpha/V_p)_2$  causes the value of  $(V_c/V_p)_2$  to be increased from its value when  $(V_{c,h} \cos \alpha/V_p)_2$  equals zero. However, as  $(V_c/V_p)_2$  becomes smaller the increase in  $(V_{c,pp}/V_p)_2$  becomes larger due to a given value of  $(V_{c,h} \cos \alpha/V_p)_2$ . As shown on Fig. 4, the change in blade row output is proportional to the change in  $(V_c/V_p)_2$ . Therefore for a given value of  $(V_{c,h} \cos \alpha/V_p)_2$ , the increase in blade row output resulting from this parameter increases with decreasing  $(V_c/V_p)_2$ .

For  $(V_c/V_p)_2$  equal to zero and for all values of  $(V_{c,pp}/V_p)_2$ , the change in  $(V_c/V_p)_2$  for a given value of  $(V_{c,h} \cos \alpha/V_p)_2$  is constant. Therefore, for a given value of  $(V_{c,h} \cos \alpha/V_p)_2$ , the change in blade row output is also constant for  $(V_c/V_p)_2$  equal to zero and all values of  $(V_{c,pp}/V_p)_2$ . The results then show that discharging the coolant with a velocity component in the direction of primary flow is most effective on the diffusion region of the blade surface.

#### Effect of Trailing-Edge Coolant Discharge on Blade Row Output

As discussed under Analyses, there are two effects on blade row output resulting from trailing-edge discharge of coolant. One effect is that the coolant flow being discharged parallel to the main gas stream contributes useful kinetic energy to the blade row. This effect is shown on Fig. 4.

The other effect of trailing-edge coolant discharge on blade row output is that the coolant flow reduces the momentum deficit occurring at the trailing edge without coolant flow and thus reduces the trailing-edge loss that occurs without coolant flow.

These results consider the trailing-edge loss without coolant flow to be the loss for a square trailing-edge geometry. The maximum trailing-edge loss that can be recovered is then considered to be the difference between the loss due to a square trailing edge with no coolant flow and the loss due to the thin slot walls occurring with coolant flow. Further, the results assume that the fraction of this maximum loss recoverable with coolant flow is proportional to the ratio of the trailing-edge discharge velocity to primary air velocity.

Based on these assumptions, the reduction in trailing edge loss due to trailing edge discharge of coolant flow is presented in Fig. 7. The results shown are for a representative ratio of slot wall thickness to total trailing edge thickness  $w/t$  of 0.25.

The results show that the trailing-edge discharge of coolant can result in significant decrease in trailing-edge loss relative to the loss without coolant flow. For instance, for a value of  $t/th$  of 0.10, which is considered representative of a cooled blade configuration and a  $(V_{c,sl}/V_{p,2})$  of 0.5, the increase in specific blade row output relative to the specific output of the uncooled blade row is about 0.5 percent due to reduced trailing-edge loss.

#### Comparison of Experimental and Analytical Results

A method for determining the effect of coolant flow on blade row output based on the described analyses together with an example of the method is presented in Appendix C.

Using the method, analytical results were obtained for blade rows on which experimental results were available. A comparison of the analytical and experimental results for these blade rows are presented in the following. All of the results shown are for a coolant temperature ratio of unity.

A comparison of experimental and analytical results for two types of stator blade transpiration coolant discharge are presented in Fig. 8. The experimental results are from Refs. 2, 4, 6, and 8. In obtaining these results, the fractional coolant flow distribution over the blade surface was obtained from available design data for one value of coolant fraction. This fractional distribution was then considered constant for other values of coolant fractions.

In Figs. 8(a) and (b), the comparative results for wire mesh shell stator blading are presented. The results on Fig. 8(a) are for coolant discharge from the complete blade surface, and the results for Fig. 8(b) are for the coolant flow blocked in the diffusion area of the blade surface. The agreement shown between experimental and analytical results is considered good for this blading.

On Fig. 8(c), the comparative results for discrete hole stator blading are presented. These results show good agreement at lower values of coolant fraction and rather poor agreement at higher values of coolant fraction. At 7 percent coolant flow the analytical method predicts about 1.5 percent less output than was obtained experimentally. This is then equal to an error of about 0.2 percent in output per percent coolant flow. The reason for the difference is not known.

On Fig. 9, the comparative results for stator blading with trailing-edge ejection are presented. The agreement for this blading is also considered good, the difference between experimental and analytical outputs being within 0.5 percent over the range of coolant flow investigated.

#### Concluding Remarks

An analysis of the effect of coolant vari-

ables on blade row output has been presented. The results show that, for a given coolant fraction, the change in output relative to the output of the uncooled blade row can be positive, negative, or zero depending on the coolant variables.

Results obtained from the analyses were compared with experimental results for stator blading having four different cases of coolant discharge, all at a temperature ratio of unity. The comparison showed good agreement for three of the cases and rather poor agreement for the other.

To further test the validity of the analytical method, more experimental data is needed, particularly at different coolant temperature ratios.

#### Appendix A Symbols

a	angle between tangent to blade surface and coolant flow velocity vector at coolant discharge location
$c_D$	independent drag coefficient for trailing-edge loss ( $c_D = 0.20$ for square trailing-edge geometry)
$\bar{e}$	kinetic energy-loss coefficient resulting from blade surface friction
g	force-mass conversion constant 32.174 ft/sec <sup>2</sup>
$\Delta h$	change in output, N; ft lb
h	specific output, N/kg; ft lb/lbm
$k_p$	coolant pressure coefficient, ratio of dynamic head of coolant flow to dynamic head of primary flow
p	absolute pressure, N/m <sup>2</sup> ; lb/ft <sup>2</sup>
R	gas constant, 287 J/kg K; 53.34 ft lb/lb R
sl	width of trailing edge slot, m, ft
T	temperature, K, °R
t	trailing edge thickness; m, ft
th	blade throat width; m, ft
V	velocity m/sec; ft/sec
w	width of trailing edge slot walls, m, ft
y	coolant fraction, ratio of coolant flow to primary mass flow
$\gamma$	ratio of specific heats

#### Subscripts:

c	coolant flow
cos a	in the direction of primary flow
e	expansion process
h	coolant hole
m	mixed

min minimum  
o absence of coolant flow  
p primary flow  
pp compression process  
sl slot  
te trailing edge  
tot total  
2 station at blade row exit before mixing  
Superscript:  
' total state

## Appendix B

### Mathematical Analyses

The mathematical expressions corresponding to the analyses described in the Analyses section are presented in two parts. First the mathematical expressions for computing the effect on blade row output of coolant discharge from all locations on the blade surface except the trailing edge are presented. Then the equations for computing the effect of trailing-edge discharge, which is a special case of coolant discharge, on blade row output are described.

As mentioned under Analyses, the analyses assume that the expansion efficiency of the primary and coolant flows are equal and that the specific output of the primary flow before mixing with the coolant flow is equal to the specific output of the uncooled blade row. These assumptions permit isentropic relationships to be used in the analyses without efficiency terms. The analyses also assumes that the coolant and primary flows mix at constant pressure equal to blade row exit static pressure.

#### Effect of Coolant Discharge Except at Trailing Edge

The analyses assumes that the following blade row data is known from blade design and heat transfer data,  $p_p$ ,  $T_p$ ,  $P_2$ ,  $P_{c,h}$ ,  $T'_{c,h}$ ,  $a$ ,  $V_{c,h}$ , and  $y$ .

The velocity of the primary flow at blade row outlet is first computed from the relation

$$V_{p,2} = \left\{ 2g \left( \frac{\gamma}{\gamma - 1} \right) RT'_p \left[ 1 - \left( \frac{P_2}{P_p} \right)^{(\gamma-1)/\gamma} \right] \right\}^{1/2} \quad (B1)$$

knowing  $V_{c,h}$  and  $T'_{c,h}$ , the static temperature of the coolant at the exit of the discharge opening is given by

$$T_{c,h} = T'_{c,h} - \frac{V_{c,h}^2}{2g} \left( \frac{\gamma - 1}{\gamma R} \right) \quad (B2)$$

and the total temperature of the coolant flow resulting from the component of coolant velocity in the direction of the primary flow (which is used for computing the effective total pressure  $P'_{c,h}$  in eq. (B4)) is equal to

$$T'_{c,h} \cos a = T'_{c,h} - \frac{V_{c,h}^2}{2g} \left( \frac{\gamma - 1}{\gamma R} \right) (1 - \cos^2 a) \quad (B3)$$

As described in the Analyses section, the effect of the coolant flow on blade row efficiency is dependent upon whether the coolant flow discharge pressure  $P_{c,h}$  is higher or lower than blade exit static pressure  $P_2$ . If  $P_{c,h}$  is greater than  $P_2$ , the coolant expands to the blade row exit static pressure, and the analyses under the following section Coolant Expanding apply.

#### Coolant Expanding

The effective total pressure of the coolant is determined from

$$P'_{c,h} = P_{c,h} \left( \frac{T'_{c,h} \cos a}{T_{c,h}} \right)^{\gamma/(\gamma-1)} \quad (B4)$$

Knowing the total effective pressure  $P'_{c,h}$  and total temperature  $T'_{c,h}$  of the coolant, the velocity of the coolant at the blade row exit is given by

$$V_{c,2} = \left\{ 2g \left( \frac{\gamma}{\gamma - 1} \right) RT'_{c,h} \left[ 1 - \left( \frac{P_2}{P'_{c,h}} \right)^{(\gamma-1)/\gamma} \right] \right\}^{1/2} \quad (B5)$$

Mixing the primary flow with the coolant flow at blade row exit at constant pressure, the mixed velocity of the two flows is computed from

$$V_{m,e} = \frac{V_{p,2} + yV_{c,2}}{(1 + y)} \quad (B6)$$

Knowing the mixed velocity, the fractional change in uncooled blade row output for the condition of  $P_{c,h}$  being greater than  $P_2$  is then given by

$$\frac{\Delta h}{h_o} = \left( \frac{V_{m,e}}{V_{p,2}} \right)^2 (1 + y) - 1 \quad (B7)$$

Having considered the case where the coolant flow expands to blade row exit, the case where the coolant flow diffuses to the blade row exit, that is, when  $P_{c,h}$  is less than  $P_2$ , is considered in the following section.

#### Coolant Diffusing

For the case where the coolant exit pressure is less than the blade row exit static pressure, as discussed in the Analyses section, it is assumed that the coolant is first compressed to stage exit static pressure isentropically. Thus, in terms of specific kinetic energy the required coolant pump work is

$$\frac{V_{c,pp}^2}{2g} = \left( \frac{\gamma}{\gamma - 1} \right) RT'_{c,h} \left[ \left( \frac{P_2}{P_{c,h}} \right)^{(\gamma-1)/\gamma} - 1 \right] \quad (B8)$$

Also, as discussed under Analyses, it is assumed that the velocity of the coolant in the direction of the primary flow is conserved during the compression. So

$$V_{c,2} = V_{c,h} \cos a \quad (B9)$$

Mixing the primary and coolant streams at constant pressure, not accounting for the pump work, the specific kinetic energy of the mixed flow is given by

$$\frac{V_{m,e}^2}{2g} = \left( \frac{V_{p,2} + yV_{c,2}}{(1+y)} \right)^2 \frac{1}{2g} \quad (B10)$$

Subtracting the pump work, the total energy of the mixed flow can be expressed by

$$\frac{V_{m,e}^2}{2g} (1+y) - y \frac{V_{c,pp}^2}{2g} \quad (B11)$$

Dividing the kinetic energy of the mixed flow by the specific kinetic energy of the uncooled flow after subtracting the specific output of the uncooled flow gives

$$\frac{\Delta h}{h_o} = \left( \frac{V_{m,e}}{V_{p,2}} \right)^2 (1+y) - y \left( \frac{V_{c,pp}}{V_{p,2}} \right)^2 - 1 \quad (B12)$$

which is the fractional change in output of the mixed flow relative to the specific output of the uncooled flow for the diffusion case.

#### Computation of Mixed Conditions at Blade Row Exit

The total effect of all flows on the specific kinetic energy of the mixed flow at blade exit and on the mixed conditions at blade exit may be obtained as follows.

The total temperature of the mixed flow is given by

$$T'_m = \frac{T'_p + \Sigma y_h T'_{c,h}}{1 + \Sigma y_h} \quad (B13)$$

and the specific kinetic energy of the total flow at blade row exit is given by

$$\frac{V_m^2}{2g} = \frac{1}{2g} \left( \frac{V_{p,2} + \Sigma y_h V_{c,2}}{1 + \Sigma y_h} \right)^2 - \frac{\Sigma y_h V_{c,pp}^2}{2g(1 + \Sigma y_h)} \quad (B14)$$

Then

$$T_m = T'_m - \frac{V_m^2}{2g} \left( \frac{r-1}{rR} \right) \quad (B15)$$

and  $P_m = P_2$  from the assumption of the analyses,

#### Effect of Trailing-Edge Coolant Flow Discharge

As discussed under the Analyses section, there are two effects on blade row output resulting from trailing-edge discharge of coolant flow. One effect is that the coolant flow, being discharged parallel to the main gas stream contributes energy to the blade row. The other effect is that the coolant flow reduces the momentum deficit occurring at the trailing edge, thus reducing the trailing-edge loss that occurs without coolant flow.

Consider first the change in output caused by the coolant energy. The fractional change in output relative to the specific energy of the uncooled blade row is given by

$$\frac{\Delta h_{te,e}}{h_o} = y \left( \frac{V_{c,sl}}{V_{p,2}} \right)^2 \quad (B16)$$

Now consider the reduction in trailing-edge loss due to trailing-edge coolant discharge as discussed under the Analyses section. Since with coolant flow there are two flows involved, the effect of trailing-edge loss on each of the flows must be considered. The effect on primary flow is considered first.

The maximum trailing-edge loss that can result occurs without coolant flow. In these analyses this maximum loss is considered to result from the trailing-edge geometry of the cooled blade without the trailing edge slot. Using a modification of the equation developed in Ref. 12, the equation for this maximum loss is written

$$\frac{\Delta h_{te,o}}{h_o} = 0.340 \left( \frac{t}{\bar{e}th} \right)^{1/3} \frac{t}{th} c_D \quad (B17)$$

The minimum trailing-edge loss that can result for the primary flow is assumed to result from one half the thickness of the two trailing-edge slot walls. Modifying Eq. (B17) to provide for the slot wall thickness gives the minimum trailing-edge loss of the primary flow

$$\frac{\Delta h_{te,p,min}}{h_o} = 0.340 \left( \frac{w}{\bar{e}th} \right)^{1/3} \frac{w}{th} c_D \quad (B18)$$

Now using the assumption of the Analyses section, that the fraction of the maximum recoverable trailing-edge loss actually recovered is proportional to  $(V_{c,sl}/V_{p,2})$ , the equation for the variation in primary air trailing edge loss with coolant flow is obtained by subtracting the trailing-edge loss recoverable from the trailing-edge loss without coolant flow.

$$\frac{\Delta h_{te,p}}{h_o} = \frac{\Delta h_{te,o}}{h_o} - \left( \frac{\Delta h_{te,o}}{h_o} - \frac{\Delta h_{te,p,min}}{h_o} \right) \frac{V_{c,sl}}{V_{p,2}} \quad (B19)$$

Having considered the trailing edge loss of the primary air, the trailing-edge loss of the coolant flow is now considered. Assuming that the coolant flow trailing-edge loss results from one half the thickness of the two slot walls, a modification of Eq. (B17) to provide for the slot geometry and the specific output of the coolant flow gives the following

$$\frac{\Delta h_{te,c}}{h_c} = 0.340 \left( \frac{w}{\bar{e}th} \right)^{1/3} \frac{w}{sl} c_D \quad (B20)$$

As shown, this equation relates the trailing-edge loss of the coolant flow to the specific output of the coolant flow. To relate the loss to the loss of the uncooled blade row, the relative energies of the coolant flow and uncooled flow must be considered. Thus



$$\frac{\Delta h_{te,c}}{h_o} = y \left( \frac{\Delta h_{te,c}}{h_c} \right) \left( \frac{V_{c,sl}}{V_{p,2}} \right)^2 \quad (B21)$$

Knowing values of  $\Delta h_{te,o}/h_o$ ,  $\Delta h_{te,p}/h_o$ , and  $\Delta h_{te,c}/h_o$ , the fractional improvement in blade row output resulting from reduced trailing-edge loss with coolant then can be computed.

$$\frac{\Delta h_{te}}{h_o} = \frac{\Delta h_{te,o}}{h_o} - \frac{\Delta h_{te,p}}{h_o} + \frac{\Delta h_{te,c}}{h_o} \quad (B22)$$

The total improvement in blade row output relative to the specific energy of the uncooled blade resulting from trailing-edge coolant ejection is then the sum of the improvements due to energy addition Eq. (B16) and the reduction in trailing-edge loss with coolant flow Eq. (B22). Thus,

$$\frac{\Delta h_{te,tot}}{h_o} = \frac{\Delta h_{te}}{h_o} + \frac{\Delta h_{te,e}}{h_o} \quad (B23)$$

#### Appendix C

##### Method for Predicting Change in Blade Row Output Due to Coolant Flow

The method for coolant flow discharge at all locations except the trailing edge is first given. Then the method for coolant discharge at the trailing edge, which is a special case, is given.

##### Coolant Flow Discharge From all Locations Except Trailing Edge

The necessary steps for making the computation are given in the example calculation. To make the computation, values must be read from three figures, 10, 11, and 12, and a relative few simple slide rule calculations made. The method for obtaining an overall blade row coolant pressure coefficient from the total change in blade row output is included in the method.

##### Coolant Flow Discharge from Trailing Edge

The necessary steps for making the computation are given in the example calculation. The example calculation is for the analytical results shown on Fig. 9.

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10. Hoerner, S. F., Fluid-Dynamic Drag, Midland Park, N.J., 1968.
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EXAMPLE OF METHOD (TRAILING EDGE COOLANT DISCHARGE)

Identification number	Quantity	Value					Comment
(1)	y	0.03	0.04	0.05	0.06	0.07	Example uses $\bar{e}$ of 0.015 from test data. Example assumes square trailing edge geometry.
(2)	t	.080	.080	.080	.080	.080	
(3)	th	.64	.64	.64	.64	.64	
(4)	w	.020	.020	.020	.020	.020	
(5)	sl	.040	.040	.040	.040	.040	
(6)	$\bar{e}$	.015	.015	.015	.015	.015	
(7)	$C_D$	.20	.20	.20	.20	.20	
(8)	$V_{c,sl}$	593	755	898	1040	1165	
(9)	$V_{p,2}$	1040	1040	1040	1040	1040	
(10)	$\% \Delta h_{te,e}/h_o = \left[ (1) \times ((8)/(9))^2 \right] \times 100$	.975	2.11	3.73	6.00	8.78	
(11)	$\% \Delta h_{te,o}/h_o = \left[ \text{Figure 13 at } (2)/(3) \right]$	1.72	1.72	1.72	1.72	1.72	
(12)	$\% \Delta h_{te,p,min}/h_o = \left[ \text{Figure 13 at } (4)/(3) \right]$	.29	.29	.29	.29	.29	
(13)	$\% \Delta h_{te,p} = \left[ (11) - (12) \right] \times \left[ (8)/(9) \right]$	.82	1.04	1.24	1.43	1.60	
(14)	$\% \Delta h_{te,c}/h_c = \left[ \text{Figure 14 at } (4)/(3) \text{ and } (4)/(5) \right]$	4.4	4.4	4.4	4.4	4.4	
(15)	$\% \Delta h_{te,c}/h_o = \left[ (10) \times (14) \right] \cdot / \cdot 100$	.04	.09	.16	.26	.39	
(16)	$\% \Delta h_{te,tot}/h_o = (10) + (13) - (15)$	1.76	3.06	4.81	7.17	9.99	

EXAMPLE OF METHOD (ALL COOLANT DISCHARGE LOCATIONS EXCEPT TRAILING EDGE)

[illegible]

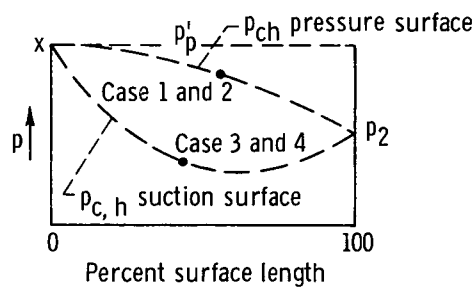
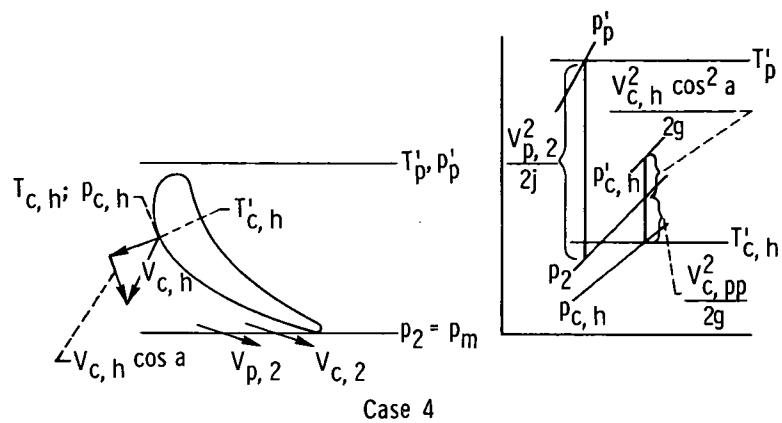


Figure 1. - Concluded.

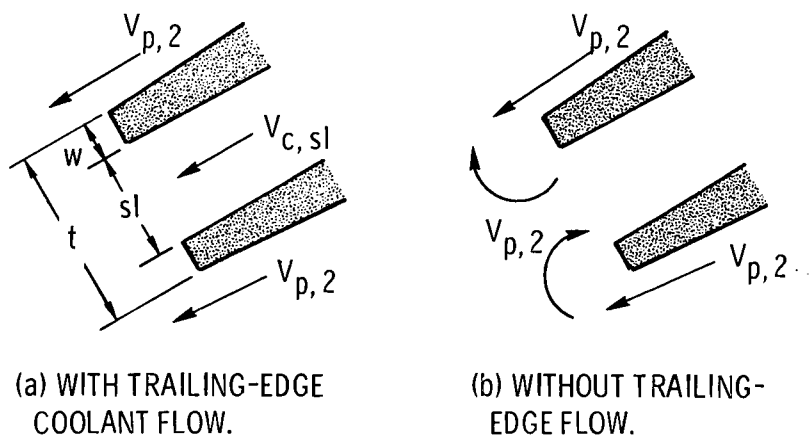
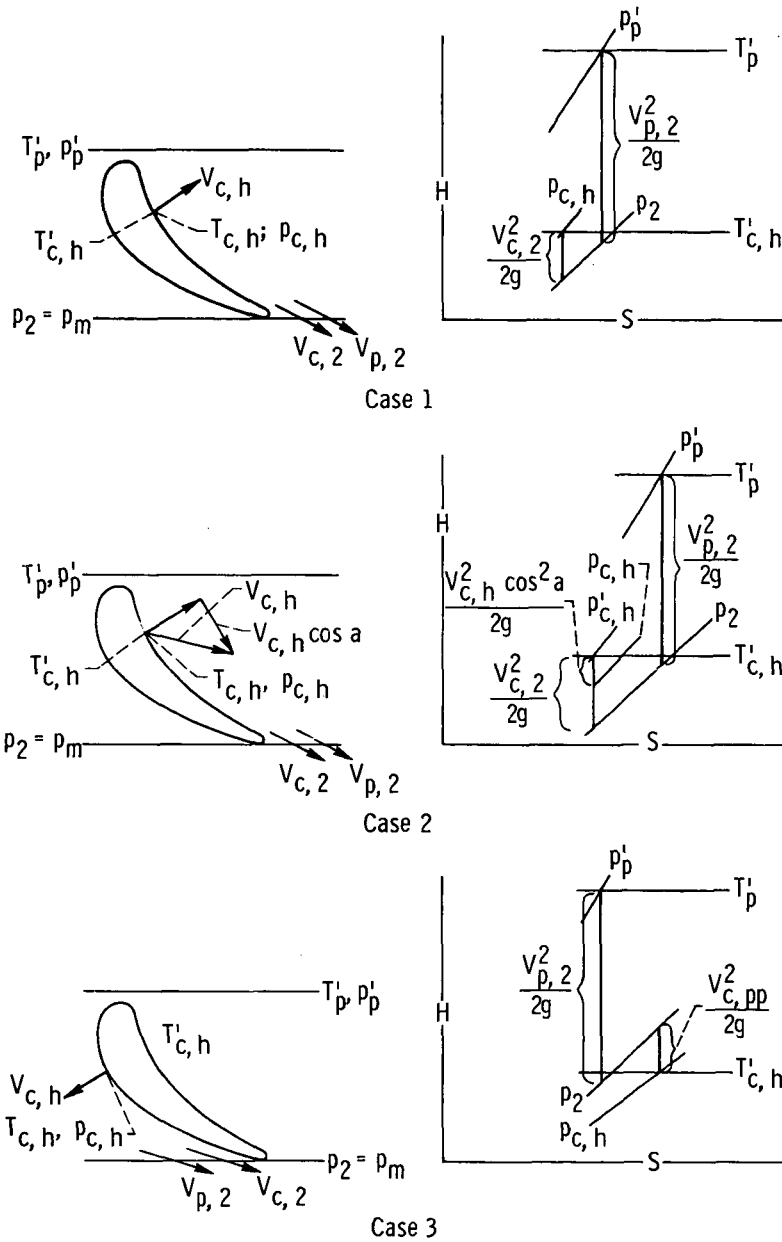


Figure 2. - Schematic of wake flow with and without trailing-edge coolant flow.



Schematics of representative cases  
of coolant discharge.

HS diagrams of representative  
cases of coolant discharge.

Figure 1. - Representative cases of coolant discharge.

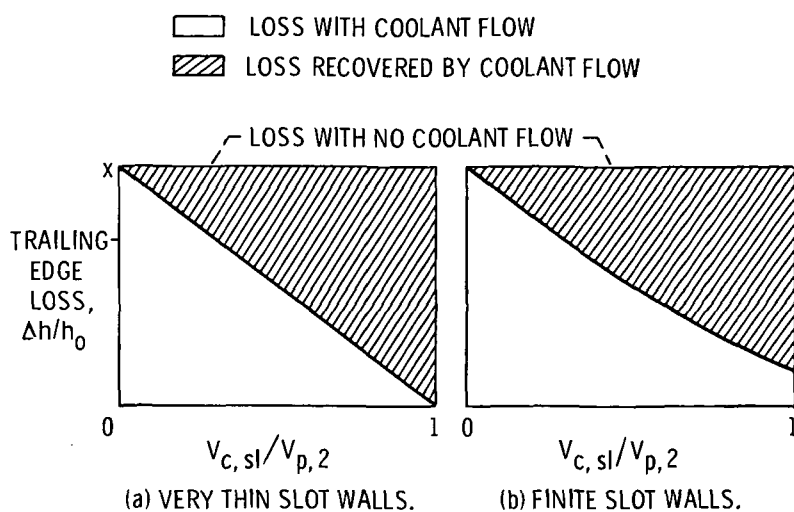


Figure 3. - General effect of trailing-edge coolant discharge on trailing-edge loss.

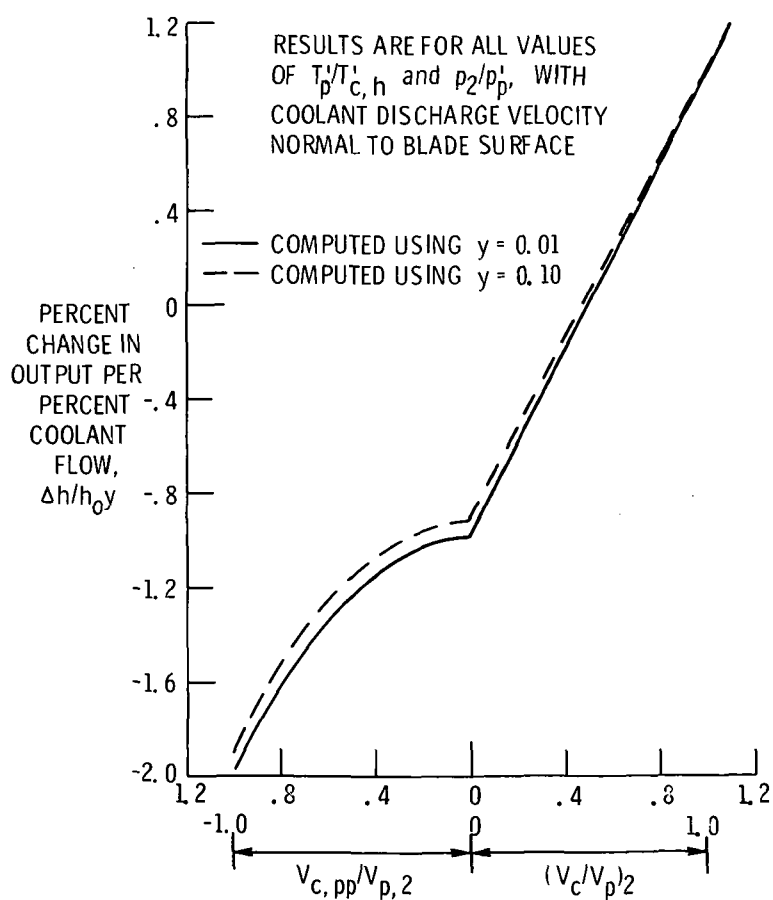


Figure 4. - Effect on blade row output of coolant velocity ratio parameters at blade row exit.

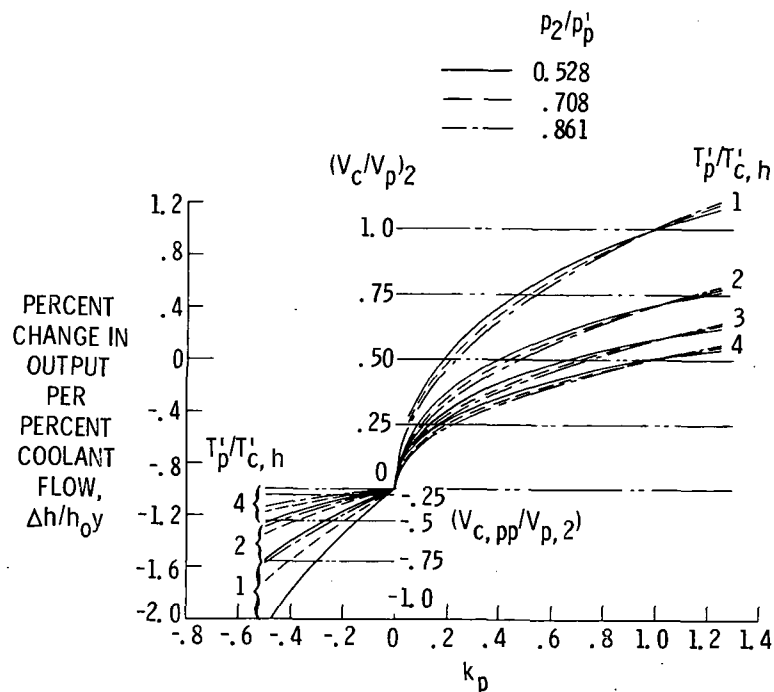


Figure 5. - Effect on blade row output of coolant temperature ratio, coolant pressure coefficient, and primary-air pressure ratio.

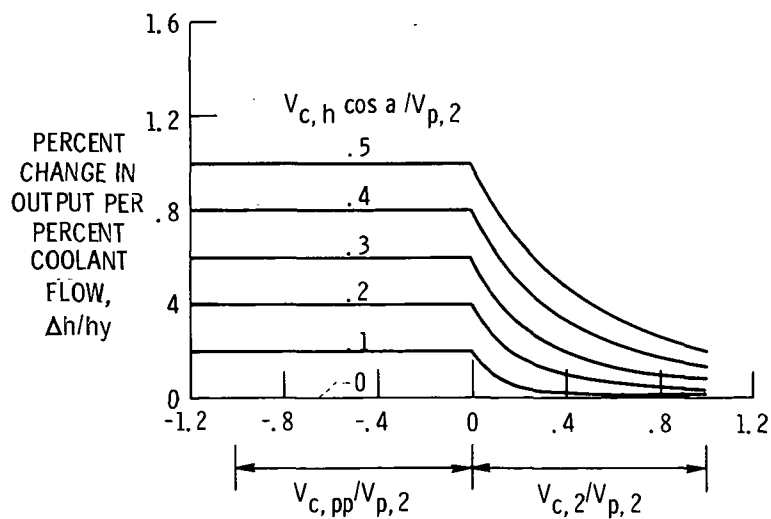


Figure 6. - Effect of coolant discharge velocity and discharge angle on blade row output.

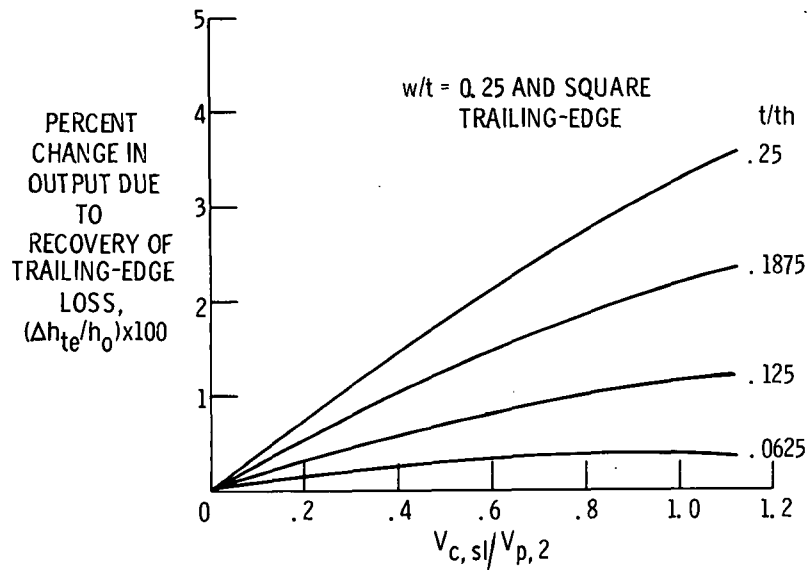
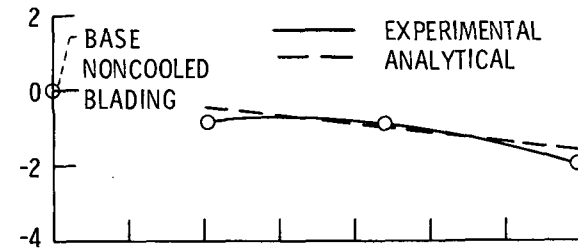
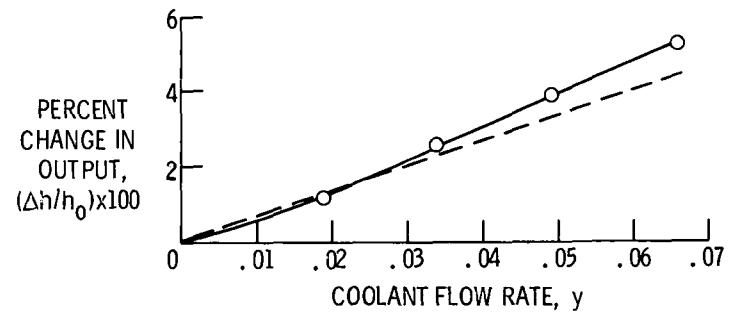


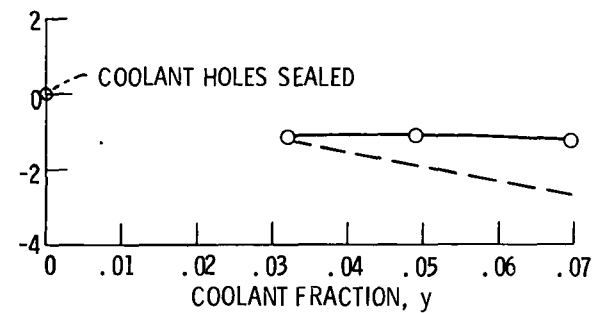
Figure 7. - Example effects of coolant flow on trailing-edge loss.



(a) WIRE MESH SHELL TRANSPIRATION STATOR BLADING.



(b) WIRE MESH SHELL TRANSPIRATION STATOR BLADING WITH COOLANT FLOW FROM BLADE DIFFUSION AREA BLOCKED.



(c) DISCRETE HOLE TRANSPIRATION STATOR BLADING.

Figure 8. - Comparison of experimental and analytical results.



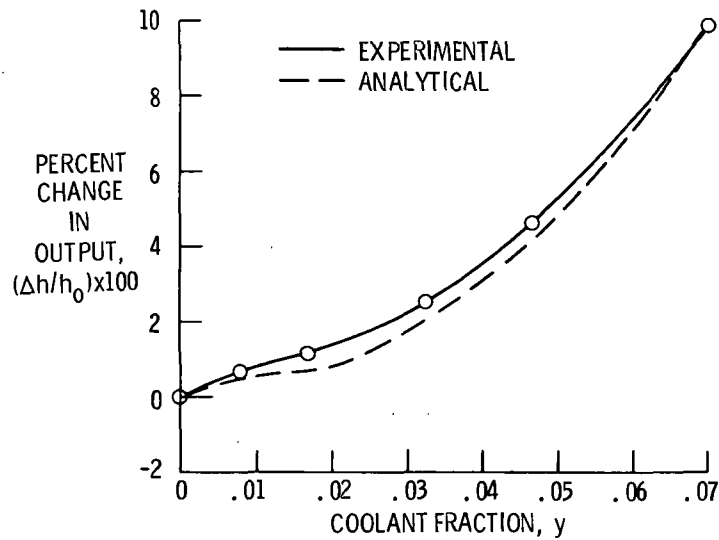


Figure 9. - Comparison of experimental and analytical results of stator blading with trailing-edge coolant ejection.

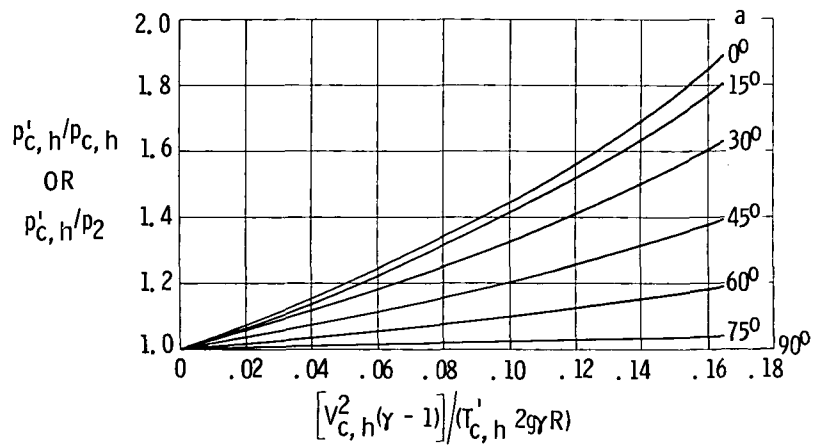


Figure 10.

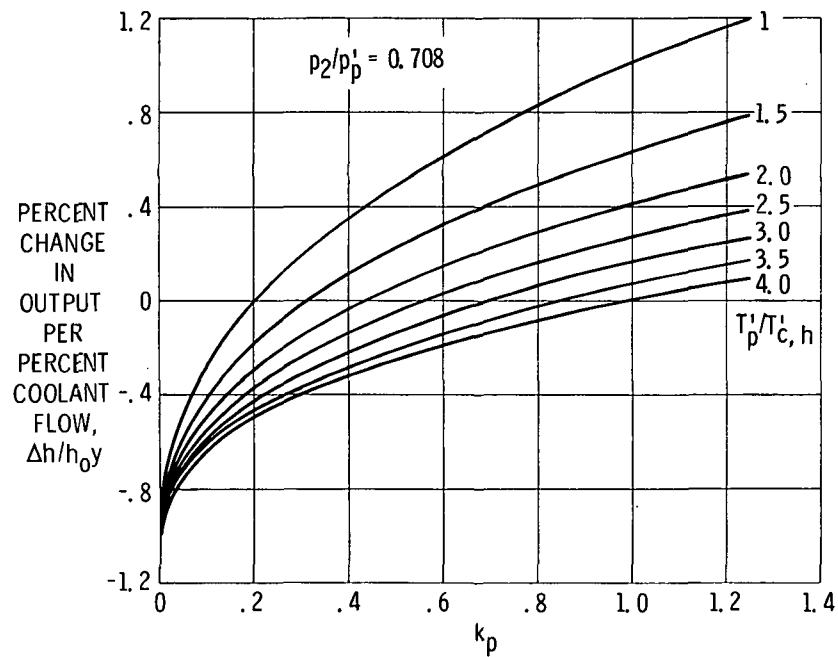


Figure 11. - Effect on blade row output of coolant flow with positive coolant pressure coefficients.

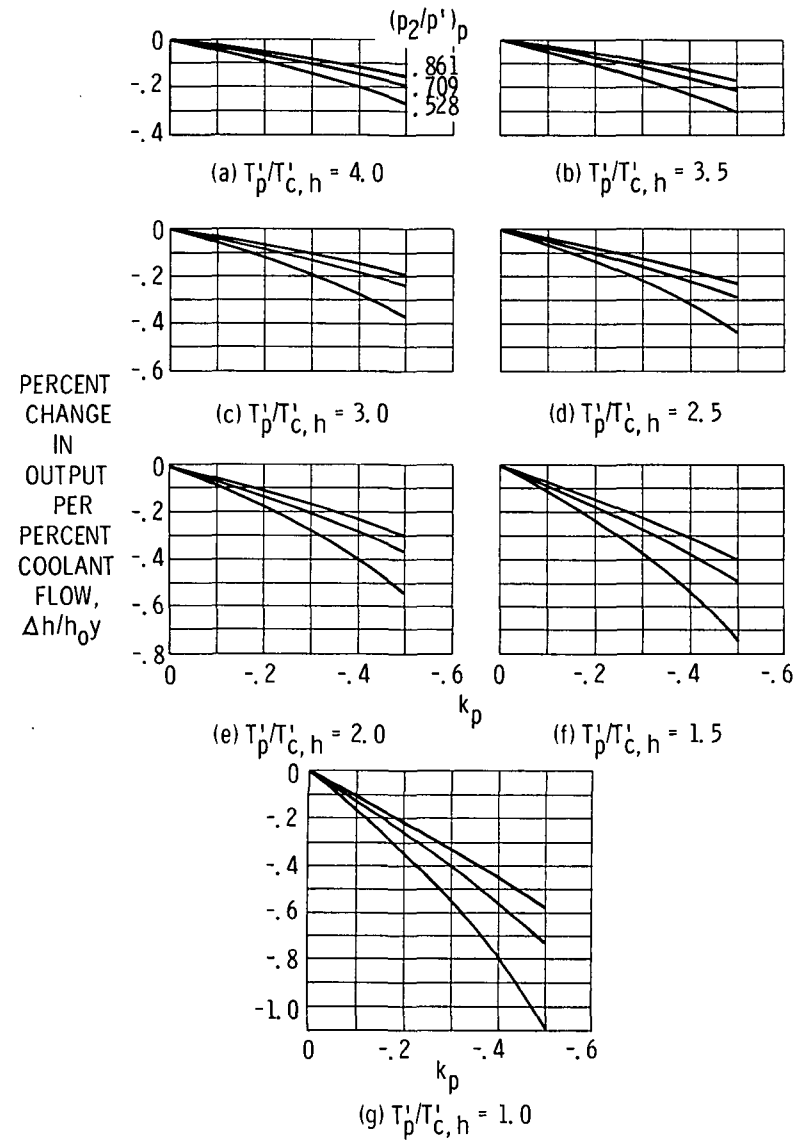


Figure 12. - Effect on blade row output of coolant pump work.

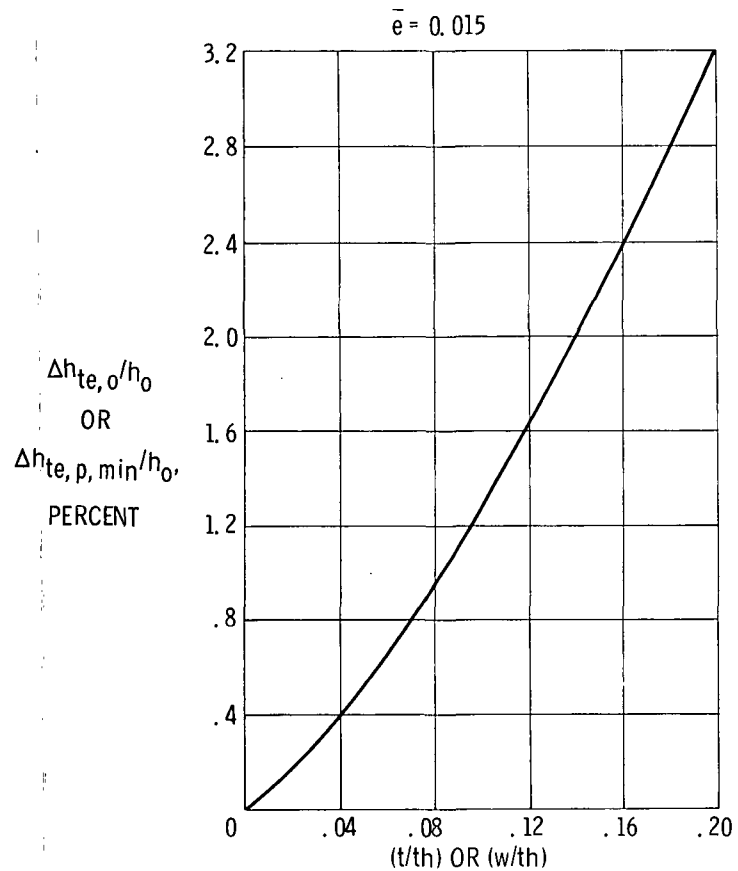


Figure 13. - Trailing-edge loss for primary flow, square trailing-edge geometry.

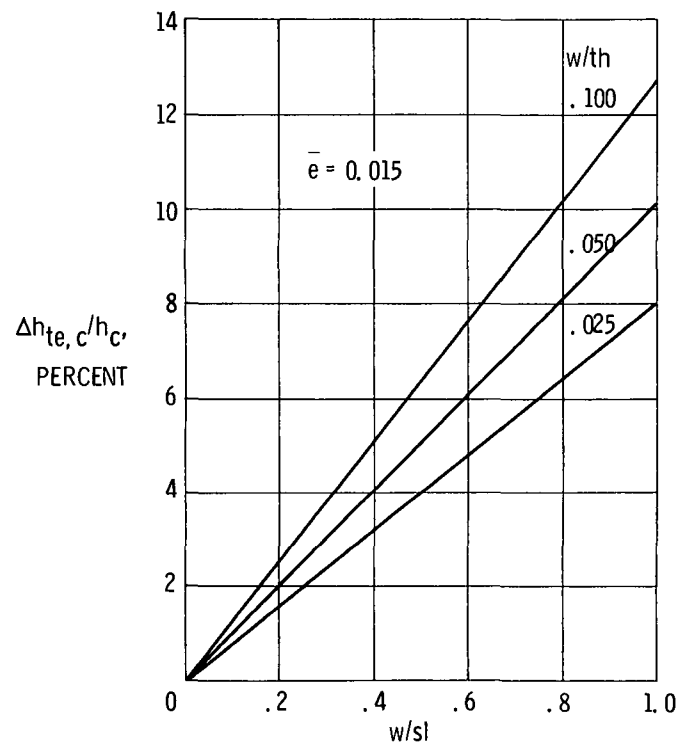


Figure 14. - Trailing-edge loss for coolant flow, square trailing-edge geometry.